TIME DEPENDENT ASSIGNMENT PROBLEM AND ITS EXTENSIONS FOR CONSTRUCTION PROJECT SCHEDULING

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ABSTRACT:

The time dependent assignment problem consists of allocating renewable resources (construction equipment, crews, or contractors, machines , jobs and persons) of limited availability to a set of activities, this perform a subtask is limited to specific intervals in time. The classical model for this problem objective is to minimize the total time or cost of completing all activities with the assumption that each activity is assigned to one particular resource at perticular period. A time-dependent task is a task requiring multiple renewable resources to perform separate subtasks simultaneously or within some predetermined margin where resource availability to perform a subtask. This paper systematizes and describes extensions of these assumptions, considering the effects of task sequence: parallel, serial and hybrid (modeled by means of network methods). This study proposes algorithms for the solution of presented models, which can be used in construction project scheduling.

KEYWORDS: assignment problem, project scheduling, mathematical modeling, renewable resources, bottleneck assignment problem

I. INTRODUCTION

The time dependent Assignment problems (selection and allocation of resources to the jobs or machines at perticular time period) are one of the primary task in construction process synchronization at particualr times. In its classical formulation n teams (i.e crews, sets of construction plant, contractors, and samples) are assigned to n activities at particualr r times, in order to minimize the total time or cost of completing all of them. Arrangement of these activities depends on technological and organizational requirements. These requirements concurrently affect the allocation constraints, such as assigning only one job to each machine at particualr periods. With respect to technological constraints, options for conducting construction processes in parallel, in series, or in a hybrid way may exist. Parallel processing consists of the simultaneous execution of activities by different resources in separate building units (work zones) at particualr times. The greatest advantage of this technique is the shortest project time span T – when compared with other approaches. The drawbacks of the parallel technique are: the lack of teams' work continuity and unleveled daily demand for building materials or plant as per duration.

Serial processing consists of performing a sequence of processes in one work zone in the seasonally. The advantage of this method is the lowest maximum level of daily employment of renewable resources and daily usage for building materials. Each activity may be realized by a different crew, but the total duration is incommensurately long. Another disadvantage, similar to the parallel technique is discontinuity of the team's work and unleveled daily consumption of resources. Hybrid processing is a combination the two previous approaches seasonally. The precedence constraints, modeled by means of network techniques, enable serial processing of some activities (on the same network path) and, simultaneously, concurrent processing of other jobs on parallel paths. The two optimization approaches are used for resource management in project networks: allocation of limited resources (in order to minimize the project makespan) and leveling resource requirements profile (in order to improve economicefficiency). This study considers different processing options for assignment problem

formulation. It has been assumed that an activity requires one resource type for its execution (crew, contractor), and the resource can perform one activity at a time with particular time periods seasonally. In the case of parallel processing, each activity has to be carried out by a different crew. With serial processing, each activity can, but does not have to be conducted by a different crew. Thus, in the case of hybrid processing,

parallel activities have to be entrusted to different crews, and activities scheduled in sequence can but do not have to be carried out by different crews. A discrete time/cost/resource function implies the representation of an activity in different modes of operation. For each activity i (construction process), a set of modes of execution is defined. Each mode, is described by the following parameters: $t_{ij} \in N$ –duration of the activity i realized by crew j and $k_{ij} \in R$ – cost of the activity i realized by crew+ Jat particular time period k. The problem consists of choosing the optimal process modes in order to minimize the project duration or cost. Binary linear programs are developed to model the assignment problem for different processing options and recommended approach solutions are presented.

II FORMULATION

1 Assignment problem formulation for parallel processing

The classical assignment problem was formulated in 1952 by D.F. Votaw and A. Orden

[1] as a type of transportation problem. Present extended study in on time dependent assignment problem. The mathematical model of the time dependent assignment problem for parallel processing can be described by an objective function minimizing the total cost of realizing processes at particualr times

MinZ=Min( )

and the following constraints:

$$\sum_{i=1} x_{ijk} = 1, \quad j = 1, 2, ..., n , k=1,2…r \quad (2)$$

$$\sum_{j=1} x_{ijk} = 1, \quad i = 1, 2, ..., n , k=1,2…r \quad (3)$$

$x_{ijk} \in \{0,1\}$, i = 1, 2, ..., n, j = 1, 2, ..., n , k=1,2…r

where:

x_{ijk} – a binary variable for modeling a decision of selecting processes' modes; the variable assumes the value of 1 if the activity i is to be executed by the resource (crew) j, and equals 0 in the other case. According to equations (2) and (3), considering equal numbers of crews and activities, each crew may be assigned only to one process and each process may be realized by only one crew at the particular time period.

H.W. Kuhn [2] in 1955 created the Hungarian method – exact algorithm for solving the model. He combined the ideas of two Hungarian mathematicians: D. König [3] and

J. Egerváry [4]. The method finds an optimal assignment for a given square cost matrix, and consist of five steps [2, 5–7]: with the considered time dependent periods.

Step 1 – Subtract the smallest entry in each row from all the entries of its row.

Step 2 – Subtract the smallest entry in each column from all the entries of its column.

Step 3 – Construct a minimal number of lines, which covers all the zero entries of the cost matrix with kij.

Step 4 – If number of covering lines is n, then it is complete. Otherwise, proceed to Step 5.

Step 5 – Determine the smallest, uncovered entry and subtract it from uncovered rows, and then add this entry to each covered column. Repeat Step 3.

The bottleneck assignment problem is one of the extensions of the classical formulation. It consist of the minimization of the project makespan – maximal duration of processes realized in parallel (each demanding a different crew):

Min z: z = max (5)

subject to the same constraints and definitions as in the classic time dependent assignment problem. In 1959 O. Gross [8] created an algorithm which is used for solving this kind problem. It may be described as follows: time dependent assignments

Step 1 – Begin with any of feasible solution, e.g. xii = 1, I = 1, 2, …, n, set of chosen entries

– B = {tij : xij = 1}.

Step 2 – Compute V = max{tij : xij = 1}.

Step 3 – Locate a cycle, which begins and ends at V as follows:

Step 3a – from V go to entry in its column with cost less than V,

Step 4 – Reverse the assignments along entries, which take part in previous steps. Let x'ij = 1 – xij for participating entries in Step 2. Proceed to Step 2.

2.Assignment problem formulation for serial processing

After completing each serial process, the renewable resources become available again and may be assigned to succeeding activities with the time periods. Therefore, the constraint (2) in the classical formulation doesn't hold in

this case. For each process, the whole set of crews is considered as the feasible solution space – in the problem of minimizing project duration as well as cost, aappropriate modes should be selected with the shortest duration for each process or with minimal cost, respectively.

Assuming that the processes sequence will be repeated continuously in d identical work zones (units), the objective function of minimizing project duration should be modified as follows:

Min z :z = (d −1)max {tijk · xijk } +

The mathematical model (converted to linear form) with binary variables for many practical instances may be solved using any commercial optimization software.

3.Assignment problem formulation for hybrid processing

A construction project can be modeled as an activity–on–node network. Precedence relations between activities are modeled by a graph G = <V, E>, directed and acyclic, with

a single initial node and a single final node, where V = {1, 2, ⋯, n} is a set of activities with dependent time periods, the edges (or arcs) E □ V × V represent precedence relations between activities. R is the set of resources − crews or contractors − available to the project. Variables si, □i ∈ V, stand for activities' start times

. Resources can be assigned to a number of processes, but not at the same time. Therefore, a set of processes' pairs J V × V can be defined, which can potentially be executed in parallel ((u, v) ∈ J □ u < v and activities u and v do not lie on the same path of the project network).

In the case that the resource j (xij = 1 □ xvj = 1) is assigned to a pair of processes (u, v) □ J , these

Processes cannot run at the same time, but have to be completed in sequence. The sequence is modeled by means of binary variables: yuv ∈{0, 1), defined for □(u, v) ∈ J. The variable yuv equals 1 if the activity u is to be completed

before activity v, and it equals 0 in the other case.

The decision making process is aimed at selecting options of resource assignment and scheduling them in such a way that project duration is minimal. To solve the problem, a mixed integer (binary) linear program is developed to model the construction project scheduling problem. The mathematical model used for this problem is described as follows:

min T : T = sn + Dn (7)

Di = ∑tijk · xijk , ∀i ∈V (8)
j∈R k∈R

∑ xijk = 1, ∀i ∈V (9)
j∈R k∈R

s1 = 0 (10)

Formulas (12) and (13) are introduced to define process start times (u, v) ∈ J. If these processes are not to be executed by the same resource j, (xij = 0 ∨ xvj = 0), both of these conditions are automatically met (M is an arbitrarily assumed, sufficiently large constant), and the processes may run concurrently. If the same resource j is assigned to them (xuj = 1□ xvj = 1), and if the variable yuv assumes the value of 1, then (in accordance with condition (12)), process v can only start after process u has been completed; in this case, condition (13) is automatically fulfilled. If the variable yuv equals 0, then process v must be completed before, u has been started – according to condition (13) and condition (12) is met automaticall

$s_i + D_i \le s_l$, ∀(i, l) ∈ E (11)

$s_u + D_u \le s_v + M \cdot (1 - y_{uv}) + M (2 - x_{uj} - x_{vj})$, ∀(u, v) ∈ J , ∀j ∈ R (12)

$s_v + D_v \le s_u + M \cdot y_{uv} + M \cdot (2 - x_{uj} - x_{vj})$, ∀(u, v) ∈ J , ∀j ∈ R (13)

$s_i \ge 0$, ∀i ∈V (14)

$x_{ijk} \in \{0,1\}$,∀i ∈V , ∀j ∈ R ∀k ∈ R (15)

$y_{uv} \in \{0,1\}$,∀(u, v) ∈ J (16)

The objective function (7) minimizes total project duration. Equation (8) determines duration Di of a activity i – it has been introduced as an auxiliary formula to simplify the formulas (7) and (11)–(13). According to condition (9), each activity can be executed in only one way – as selected from available options. Execution

of the first activity of the project (i.e. a activity that has no predecessors) starts at the moment of 0 (10). Condition (11) defines the successors' start dates as "not earlier than their predecessors have been completed with time dependent condition".

The problem of project cost minimization is trivial and in the optimal solution the crews with the lowest cost for each process are assigned to realize it. Because the project duration for this assignment may be unacceptably long, the objective could be modified as follows

Min k: k = (17)

and the following constraint needs to be added to assure not exceeding the

deadline Td.

$s_n + D_n \le T_d$ (18)

III. EXAMPLE

Durations (in days) and costs (in monetary units) for completing a particular processes by particular crews in an example project is presented in matrices T and K respectively.

□9 7 6 5 4□
6 5 8 6 4 □

T = 3 5 2 5 5 □

□4 4 4 3 4□
6 5 8 7 6 □□ ,

□1 0 0 0 0 □
□0 0 1 0 0 □
□
X1 = □0 0 0 0 1□
I □
□0 1 0 0 0 □
□□0 1 0 1 0 □

The hybrid precedence relations of the processes is settled according to the technological constraints and shown in Fig. 2.

Fig. 2.

There are two optimal solutions of the assignment problem for parallel processes (with minimal project cost of 19 monetary units) obtained using the Hungarian algorithm:

□1 0 0 0 0 □
□0 0 1 0 0 □
□□
X1 = □0 1 0 0 0□ ,
I □□
□0 0 0 0 1 □
□ □0 1 0 1 0 □□

and

□0 0 0 1 0□
□ 0 0 0 0 1 □
□□
X1 = □0 0 1 0 0 □ .
II □□
□1 0 0 0 0□
0 1 0 0
0 □□

The Hungarian algorithm prompts that there are three different optimal solutions obtained for the original assignment problem with minimal

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sum of processes' durations (20 days):

The solution 1 is also obtained by means of the algorithm by Gross. The minimal.duration for the bottleneck assignment problem is 5 days. Because the solutions for both the bottleneck and the original assignment problems are the same, so this is also the optimal crew assignment for the sequence of processes repeated in different units (regardless of the units number).

The optimal solution for crew assignment processes with hybrid precedence constraints are as follows:

The minimal project duration for a given network is 10 days. Solution of the mathematical model (7)–(16) based on the above input values was found by means of LINGO 12.0 Optimization Modelling Software by Lingo Systems Inc.

IV. CONCLUSION

Scheduling with the allocation of constrained resources, particularly for skilled labor, is a major challenge for almost all construction projects with time dependent constraint. Most existing techniques for project scheduling consider a single–skilled resource strategy where each worker or crew is assumed to be of one particular trade. This strategy may lead to inefficiencies in labor utilization, which can also be reflected in increased project durations and unnecessary costs. In practice each construction contractor or crew can possess several skills at different proficiency levels,

i.e. they are able to perform more than one type of work (construction process), each at specified times and costs. The assumption that each worker may possess multiple skills which could allow them to participate in any activity that fits one of their skills, can improve project efficiency in terms of project cost and duration with considered time durations.

The resources assignment models presented in the paper can help managers in determining a strategy for crew or bids selection.

For the case of parallel processing there are exact algorithms available for solving assignment problems with low computational effort. For hybrid processing options, the problems analyzed in this paper can be considered as an extension of the Resource–Constrained Project Scheduling Problem (RCPSP) which is NP–hard. Exact algorithms may be not efficient to solve complex practical problems with considered time dependent constraint , therefore developing heuristic solving procedures is recommended for further research.

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